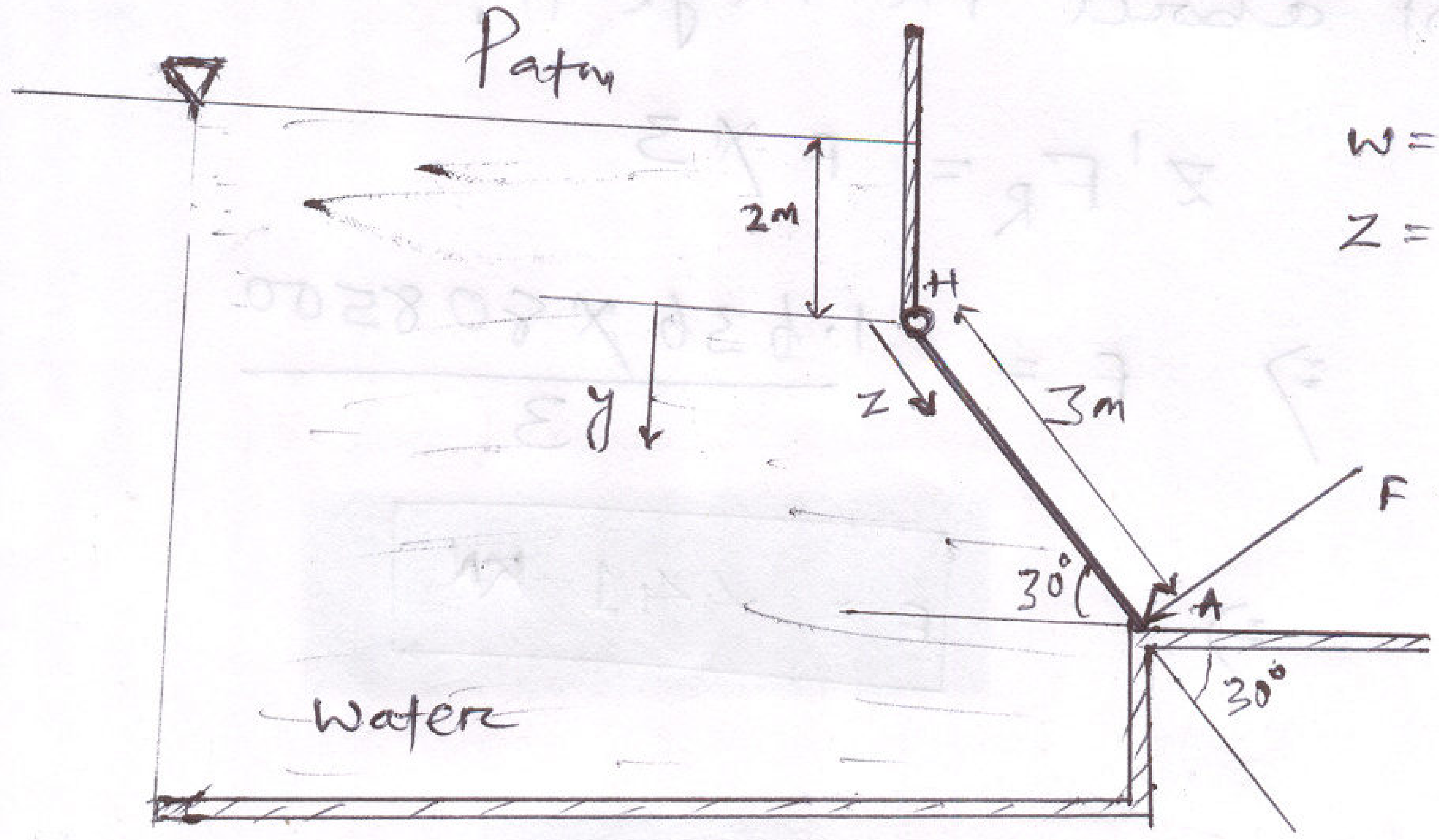
Question. 1



W = Width of gate

Z = Normal to the plane of paper

$$y = z \sin 30^\circ = \frac{z}{2}$$

Resultant force on 'A'

$$= F_R = W \int_{z=0}^{z=3} (2+y)\, \rho g \, dz$$

$$= \rho g W \int_{z=0}^{z=3} \left(2 + \frac{z}{2}\right) dz$$

$$= 10^3 \times 9.8 \times 10 \left[2z + \frac{z^2}{4}\right]_{z=0}^{z=3} = 10^4 \times 9.8 \times (6+$$

$$\Rightarrow \boxed{F_R = 808500\ N}$$

Line of action of Resultant force (z')

$$\Rightarrow z' F_R = \int_{z=0}^{z=3} z P \, dz = \rho g W \int_{z=0}^{z=3} z\left(2 + \frac{z}{2}\right.$$

$$= \rho g W \left[z^2 + \frac{z^3}{6}\right]_{z=0}^{z=3}$$

$$= 10^3 \times 9.8 \times 10 \left[9 + \frac{27}{6}\right]$$

$$\Rightarrow z' \times 808500 = 10^4 \times 9.8 \times 13.5$$

$$\Rightarrow \boxed{z' = 1.636\ m}$$

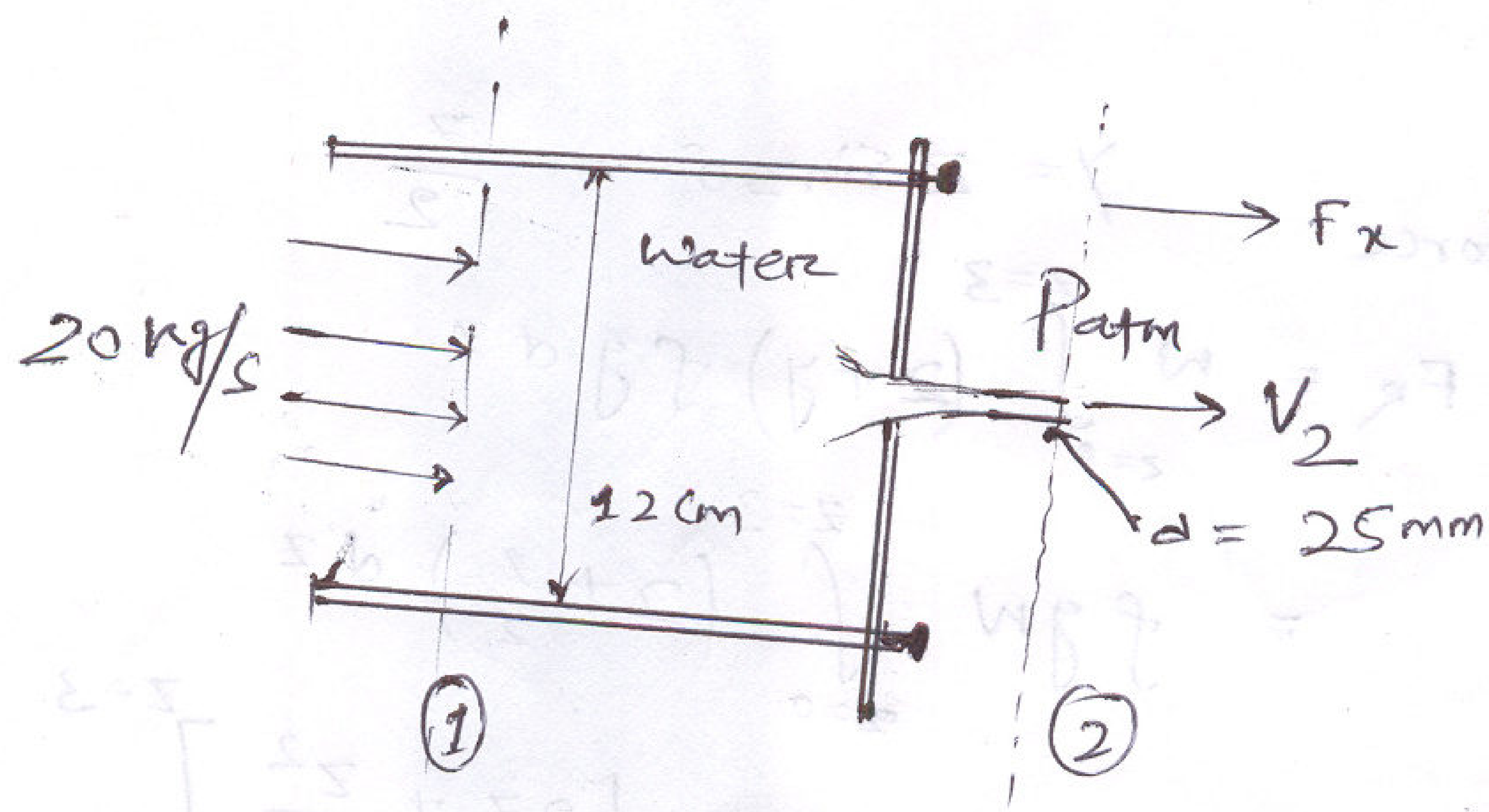
Moment about the Hinge H.

$$z' F_R = F \times 3$$

$$\Rightarrow F = \frac{1.636 \times 808500}{3}$$

$$\Rightarrow \boxed{F = 441 \text{ KN}}$$

Question. 2



F_x = 'X'- componet of force that must be applied to the plate.

Water flow rate = $\dot{Q} = 20 \text{ kg/s} = 0.02 \text{ m}^3/\text{s}$

At Section 1

$$\text{Area} = A_1 = \frac{\pi d^2}{4} = \pi/4 \times (0.12)^2$$

$$\text{Velocity} = V_1 = \frac{\dot{Q}}{A_1} = \frac{0.02}{\pi/4 \times (0.12)^2} = 1.77 \text{ m/s}$$

Doing the mass balance at section '1' & '2'.

$$\Rightarrow V_1 A_1 = V_2 A_2$$

$$\Rightarrow V_2 = \frac{V_1 A_1}{A_2} = \frac{\dot{Q}}{A_2}$$

where $A_2 = \pi/4 \times (0.0$

$$= \frac{0.02}{\pi/4 \times (0.025)^2}$$

$$\Rightarrow V_2 = 40.74 \text{ m/s}$$

By Momentum Balance

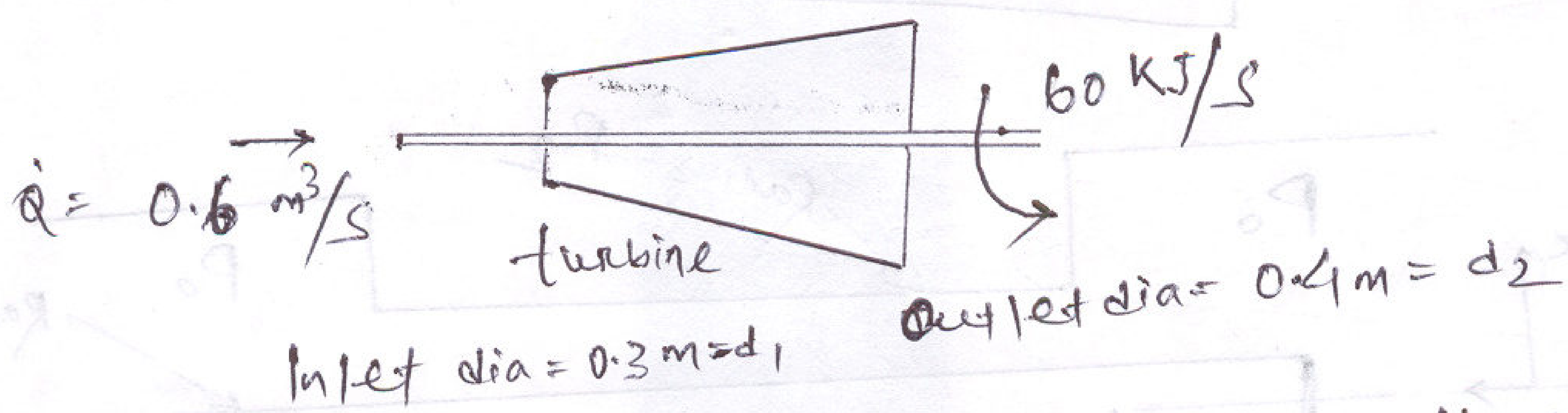
$$F_x + 800 \times 10^3 \times \pi/4 \times (0.12)^2$$

$$= -20 \times 1.77 + 20 \times 40.74$$

$$\Rightarrow \boxed{F_x = -8268.4 \text{ N}}$$

So, Force must be applied in the negative x-direct

Question-3:



Here $z_1 = z_2$

Doing Energy Balance:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{z_1 g} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{z_2 g} + W_{shaft} + \cancel{W_{loss}}$$

Neglect

$$\Rightarrow \frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + W_{shaft} \quad \text{---} (1)$$

$$V_1 = \frac{0.6}{\pi/4 \, (0.3)^2} = 8.48 \text{ m/s}$$

$$V_1 A_1 = V_2 A_2 \Rightarrow V_2 = V_1 \left(\frac{A_1}{A_2}\right) = V_1 \frac{d_1^2}{d_2^2}$$

$$W_{shaft} = \frac{\dot{W}_s}{\dot{m}} = \frac{\dot{W}_s}{\rho Q}$$

So, putting all this values in eq-① and rearranging.

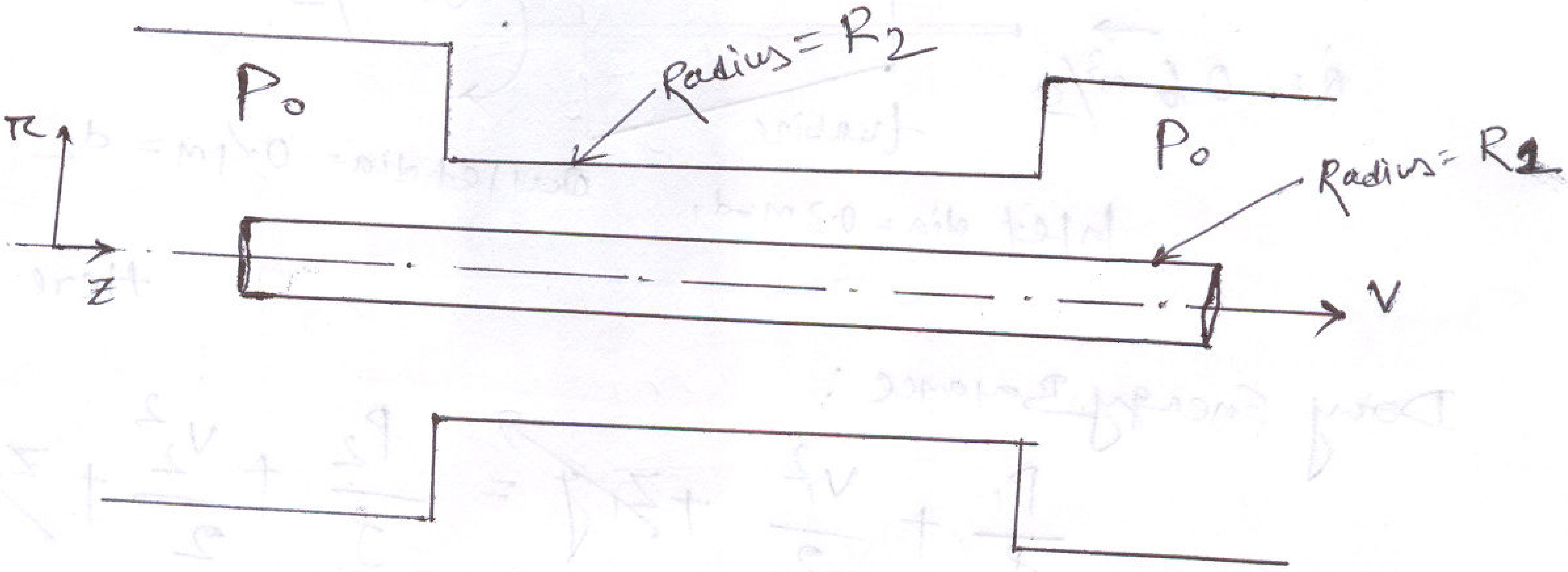
$$P_1 - P_2 = \frac{\dot{W}_s}{\rho \dot{Q}} + \frac{1}{2}\rho V_1^2\left(\frac{d_1^4}{d_2^4} - 1\right)$$

$$= \frac{60\times10^3}{0.6} + \frac{1}{2}\times10^3\times(8.48)^2\times\left[\left(\frac{0.3}{0.4}\right)^4 - 1\right]$$

$$\Rightarrow \quad P_1 - P_2 = 75421\,\text{Pa}$$

So, $\boxed{\text{Pressure drop} = 75.42\ \text{kPa}}$

Question-4



Given: $\frac{dp}{dz} = 0$; Steady and fully-developed flow.

~~So~~ From the given conditions flow is axisymmetric.

From equation of continuity; $V_r = \text{Constant}$ | As $V_\theta = 0$

Fully developed flow, $\frac{\partial V_z}{\partial z} = 0$

$$\Rightarrow \quad V_z = V_z(r)$$

From z-momentum equation in cylindrical coordinates

$$\rho\left[\underbrace{\cancel{\frac{\partial V_z}{\partial t}}}_{0\ \text{steady state}} + \underbrace{\cancel{V_r\frac{\partial V_z}{\partial r}}}_{0\ \text{from continuity}} + \underbrace{\cancel{\frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}}}_{0\ \text{axisymmetric}} + \underbrace{\cancel{V_z\frac{\partial V_z}{\partial z}}}_{0\ \text{(fully developed)}}\right] = \underbrace{\cancel{-\frac{\partial p}{\partial z}}}_{\text{given Po in}} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}}}_{0\ \text{(axisymmetric)}} + \cancel{\frac{\partial^2 V_z}{\partial z^2}}\right]$$

$$\Rightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial V_z}{\partial r}\right] = 0$$

$$\frac{\partial}{\partial r}\left[r\frac{\partial V_z}{\partial r}\right] = 0 \Rightarrow r\frac{\partial V_z}{\partial r} = C_1 \Rightarrow \frac{\partial V_z}{\partial r} = \frac{C_1}{r}$$

$$\Rightarrow \quad V_z = C_1 \ln r + C_2$$

Boundary conditions:

$$V_z(r = R_1) = V$$

$$V_z(r = R_2) = 0$$

$$\Rightarrow \quad V = C_1 \ln R_1 + C_2$$

$$0 = C_1 \ln R_2 + C_2$$

(−)

$$V = C_1 \ln R_1 - C_1 \ln R_2 = C_1 \ln\left(\frac{R_1}{R_2}\right)$$

$$\Rightarrow \quad C_1 = V \Big/ \ln\left(\frac{R_1}{R_2}\right)$$

$$\text{So, } C_2 = -C_1 \ln R_2 = -\frac{\ln R_2}{\ln(R_1/R_2)} V$$

$$V_z = C_1 \ln r + C_2$$

$$= \frac{V}{\ln\left(\frac{R_1}{R_2}\right)} \ln r - \frac{V \ln R_2}{\ln\left(\frac{R_1}{R_2}\right)}$$

$$\Rightarrow \quad \boxed{V_z(r) = \frac{V}{\ln\left(\frac{R_1}{R_2}\right)} \ln\left(\frac{r}{R_2}\right)}$$

Shear stress exerted by the fluid on the wire:

$$\tau_{rz} = \mu \frac{\partial V_z}{\partial r}\bigg|_{r=R_1}$$

$$= \frac{\mu V}{R_1} \cdot \frac{1}{\ln\left(\frac{R_1}{R_2}\right)}$$

where $\frac{\partial V_z}{\partial r} = \frac{C_1}{r}$

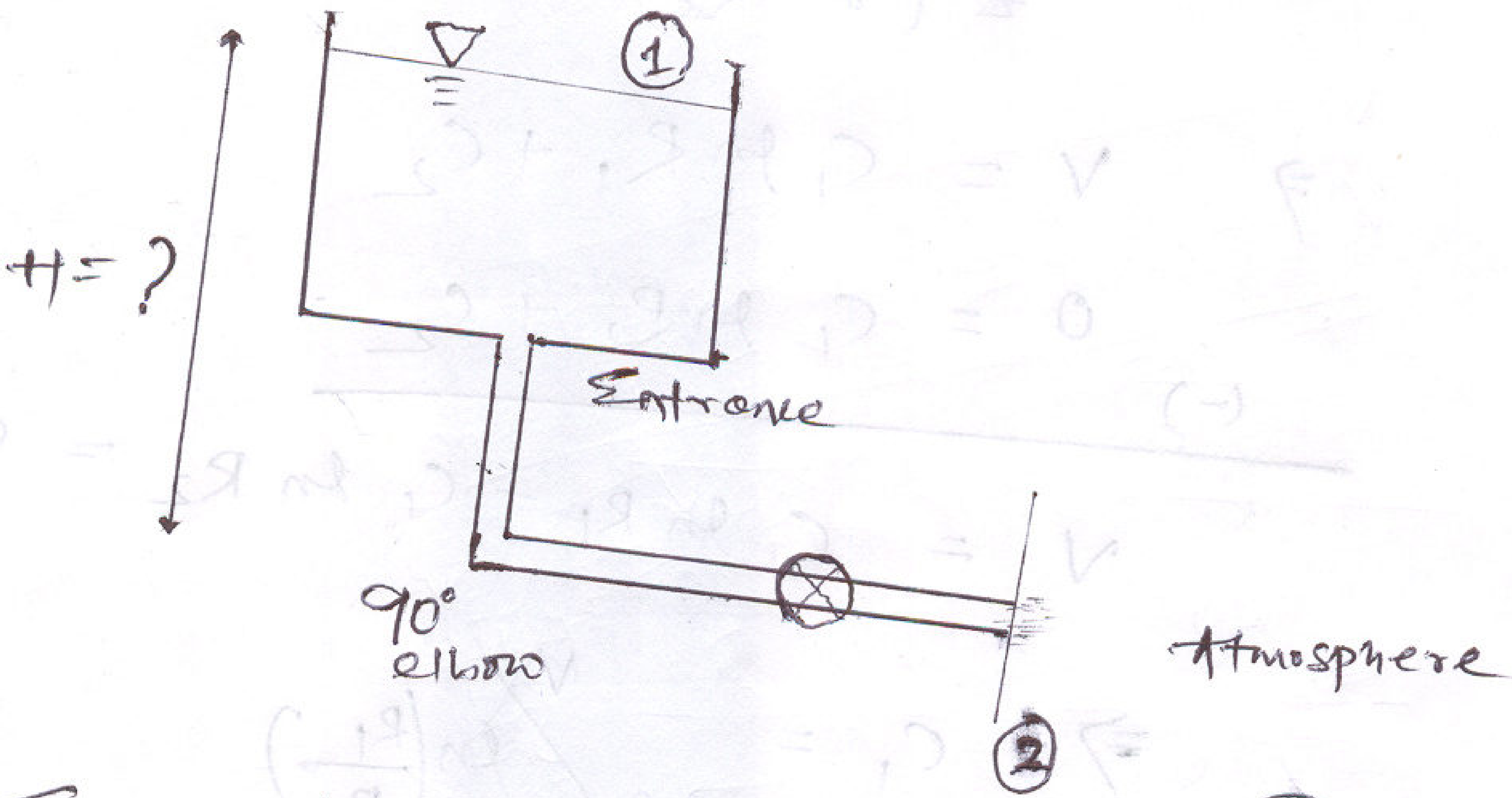
$$= \frac{1}{r} \cdot \frac{V}{\ln\left(\frac{R_1}{R_2}\right)}$$

Force per unit length on the wire:

$$= \tau_{rz} \cdot (2\pi R_1)$$

$$\Rightarrow \boxed{F = \frac{2\pi\mu V}{\ln\left(\frac{R_1}{R_2}\right)}}$$

Question-5:



Applying Energy balance between 1 and 2

$$\left(\frac{P_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{P_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$= f\frac{L}{D}\frac{\bar{V}_2^2}{2} + \frac{\bar{V}_2^2}{2}\sum_i K_i$$

Here $\alpha \simeq 1$; $\bar{V}_1 \simeq 0$; $P_1 = P_2 = P_{atm}$

$$z_1 - z_2 = H$$

$$\Rightarrow H = \frac{\bar{V}_2^2}{2g} + f\frac{L}{D}\frac{\bar{V}_2^2}{2g} + \frac{\bar{V}_2^2}{2g}(0.5+0.9+0.3)$$

given values: $Re = 10^5$, $D = 5\times10^{-2}$ m

$\rho_{water} = 10^3$ kg/m^3, $\mu = 10^{-3}$ kg/ms

for $Re = 10^5$ f = friction factor = 0.018 (Smooth Pipe)

So, $\bar{V}_2$ is

$$\Rightarrow \quad 10^5 = \frac{5\times10^{-\cancel{5}}\times10^3\times\bar{V}_2}{10^{-3}}$$

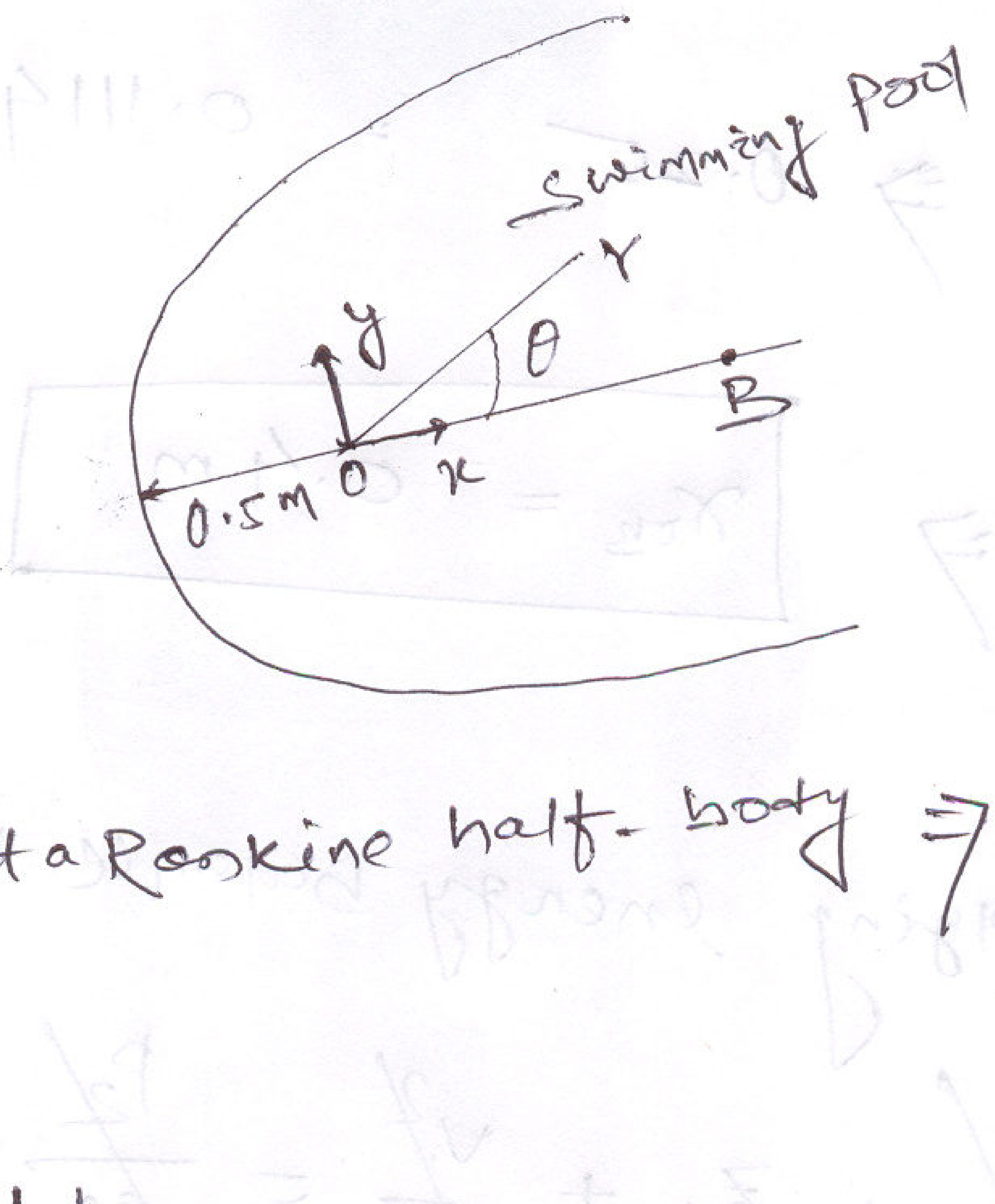
$$\Rightarrow \quad \bar{V}_2 = 2\,m/s$$

$$H = \frac{V_2^2}{2g}\left[1+0.018\times\frac{1}{5\times10^{-2}}+0.5+0.9+0.3\right]$$

$$= \frac{4}{2\times9.8}\times3.06$$

$$\Rightarrow \quad \boxed{H = 0.624\,m}$$

Question. 6



To get a Rankine half-body ⇒ Superposition of a Source of uniform flow

$$\psi = Uy + m\theta = Uy + m\tan^{-1}(y/x)$$

$$\psi = Ur\sin\theta + m\theta \qquad \text{where } r = \sqrt{x^2+y^2}$$

$$u = \frac{\partial\psi}{\partial y} = U + m\frac{x}{x^2+y^2} = U + \frac{m}{r}\cdot\frac{x}{r}$$
$$= U + \frac{m}{r}\cos\theta$$

$$V = -\frac{\partial\psi}{\partial x} = \frac{m}{r}\sin\theta$$

Stagnation point: ("nose of the half body"): $U=0, V=$

$$\Rightarrow \theta = \pi, \quad r = \frac{m}{U} \qquad \text{let } a = \frac{m}{U}$$

or $x = -m/U, \; y = 0$

Given $a = 0.5\,m \quad \Rightarrow \frac{m}{U} = 0.5$

$$m = \frac{\theta}{2\pi b} = \frac{0.35}{2\pi(1)} = 0.0557 \; m^2/s$$

$$U = \frac{m}{a} = \frac{0.0557}{0.5} = 0.1114 \; m/s$$

Given $V_B = 0.25 \; m/s = U + \frac{m}{r_{OB}}\cos 0$

$$\Rightarrow 0.25 = 0.1114 + \frac{0.0557}{r_{OB}}$$

$$\Rightarrow \boxed{r_{OB} = 0.4\,m}$$

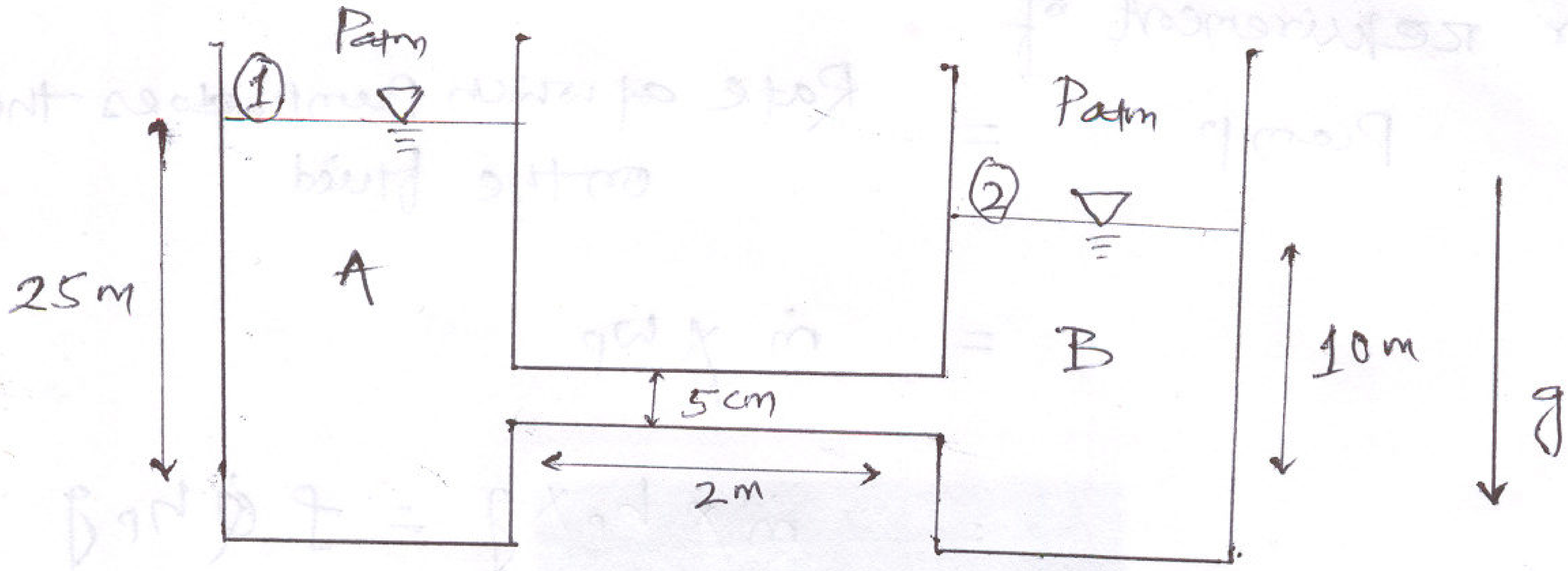
Question: 7

(a) Applying energy balance between section (1) an

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_L$$

Here $P_1 = P_2 = P_{atm}, \quad V_1 = V_2 \simeq 0$

$$\Rightarrow (z_1 - z_2) = h_L = 5.4\frac{V_{tube}^2}{2g}$$



$$\Rightarrow \quad V_t = \sqrt{\frac{15 \times 2 \times 9.8}{5.4}}$$

$$\boxed{V_t = 7.38 \ m/s}$$

(b) Now a pump is installed in the pipe. So applying energy energy balance at two section 1 and 2

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + Z_1 = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + Z_2 - h_p + h_L$$

where h_p = pump Head

$$= \frac{W_p}{g}$$ → Rate at which pump does work on the fluid / mass flow rate

$$\Rightarrow \quad Z_1 - Z_2 = \frac{5.4 \, V_t^2}{2 \times 9.8} - h_p$$

Here $Z_1 - Z_2 = 15m$, $V_t = 10 \ m/s$

$$\Rightarrow \quad \boxed{h_p = 12.55 \ m}$$

Power requirement of Pump = Rate at which pump does the work on the fluid

$= \dot{m} \times w_p$

$= \dot{m} \times h_p \times g = \rho \dot{Q} h_p g$

$= 10^3 \times \frac{\pi}{4} (0.05)^2 \times 10 \times 12.55 \times 9.8$

$\therefore$ Rate at which pump does work on fluid $= \boxed{2415\ J/sec}$

(c) Flow is now reversed (i.e from B to A)

$\Rightarrow Z_2 = Z_1 - h_p + h_L$

$\Rightarrow h_p = h_L + (Z_1 - Z_2)$

$= \frac{5.4 \times 10^2}{2 \times 9.8} + 15 = 42.55m$

$w_p = h_p g$; $\dot{w}_p = \dot{m} w_p = \dot{m} h_p g$

$= \rho \dot{Q} h_p g$

$= \boxed{8187\ J/s}$

(d) Application of Bernoulli between point (1) and (2) results in

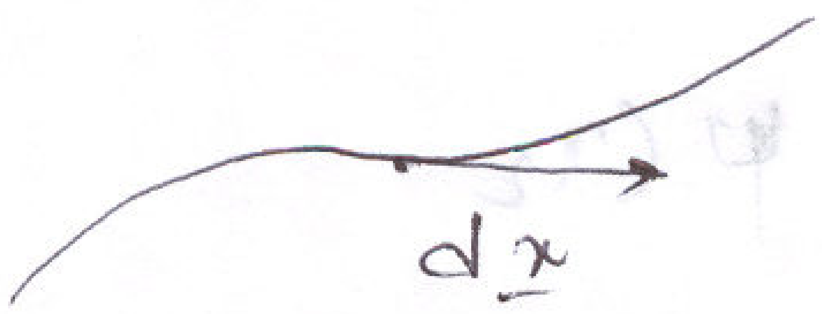
$g z_1 = g z_2 \Rightarrow$ This is a contradiction

Since here $Z_1 \neq Z_2$

The reason for this contradiction is that Bernoulli e neglects all the losses; which is not true here. The gravitational potential energy is converted to internal energy by viscous losses h_e; which is set to zero in Bernoulli equation.

Question-8

(a) $\underline{v} \times d\underline{x} = 0$ along a stream line



$$\Rightarrow \begin{vmatrix} i & j & k \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$

$$\Rightarrow \boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

(b) For 2-D flow: $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$

$$\psi = \psi(x, y)$$

$$\Rightarrow d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

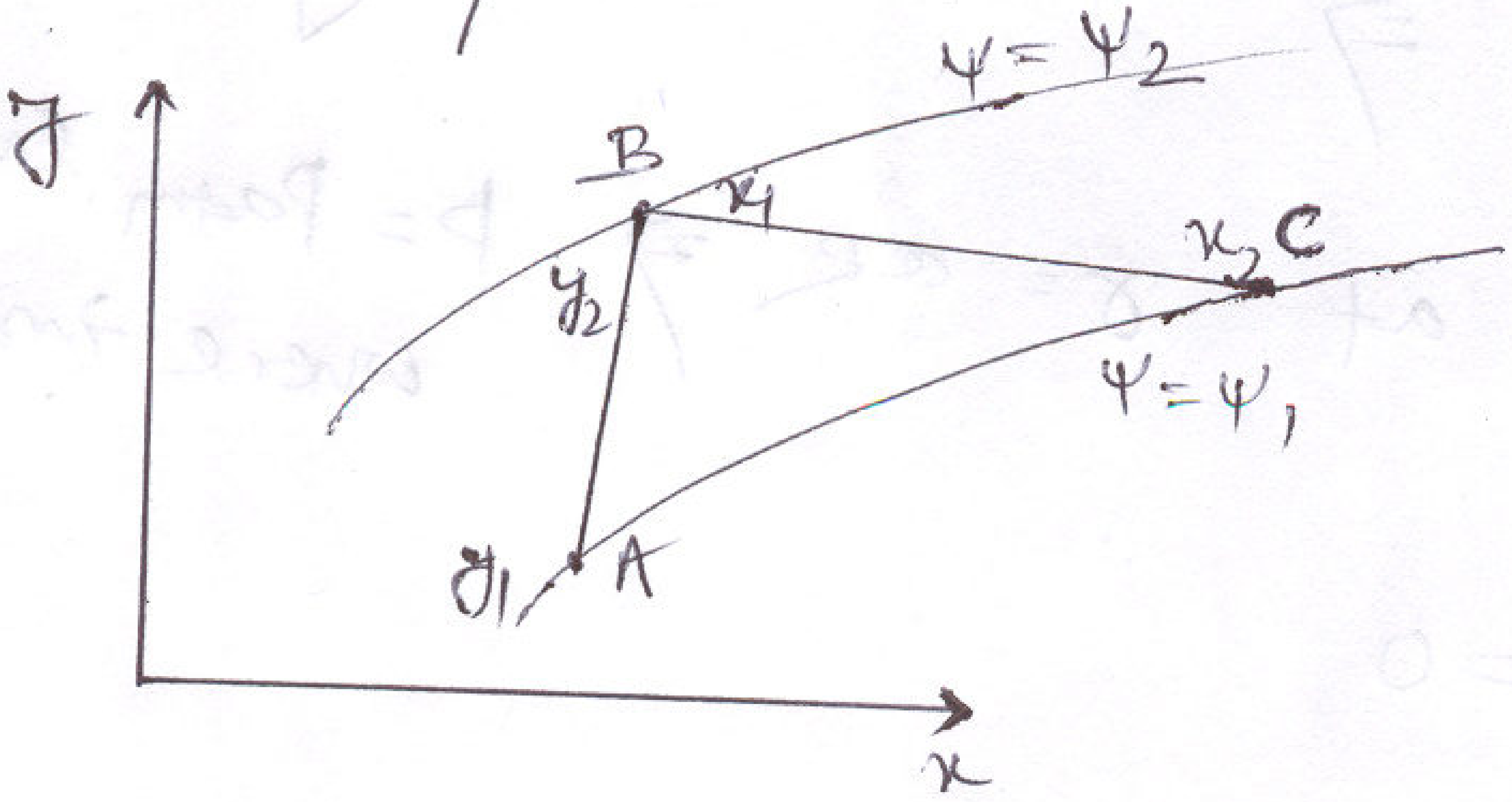
$$= -v\,dx + u\,dy$$

But, $\frac{dx}{u} = \frac{dy}{v} \Rightarrow u\,dy - v\,dx = 0$

So, $d\psi = 0$ along a streamline

$\Rightarrow \psi$ is a $\underline{\text{constant}}$ along a stream-line.

(c)



Along AB Q (per width)

$$= \int_{y_1}^{y_2} u\,dy$$

$$= \int_{y_1}^{y_2} \frac{\partial \psi}{\partial y} dy$$

Since x = constant at AB

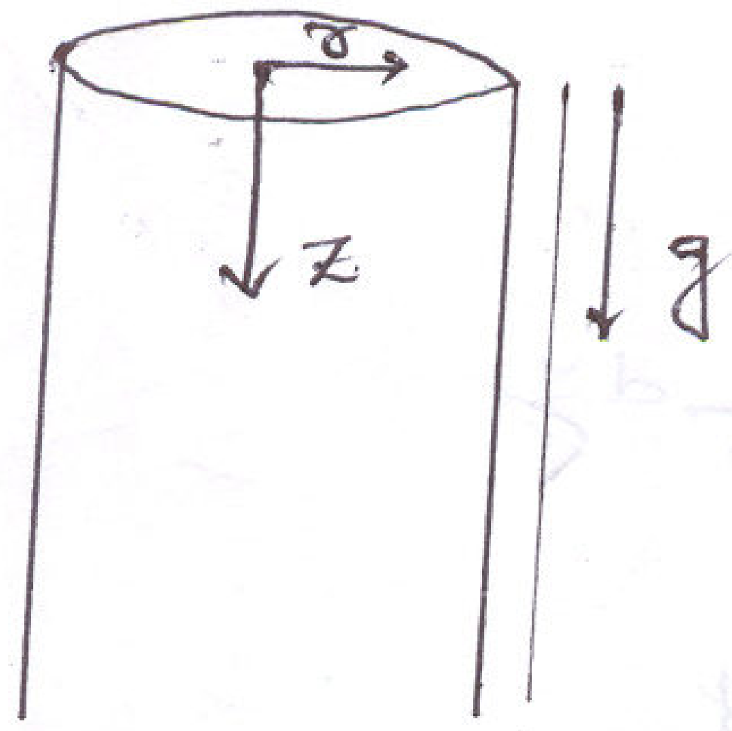
$$Q = \int_{y_1}^{y_2} d\psi = \psi(y_2) - \psi(y_1) = \psi_2 - \psi_1$$

Along BC, $Q = \int_{x_1}^{x_2} v\,dx = \int_{x_1}^{x_2} -\frac{\partial \psi}{\partial x}\,dx = -\int_{x_1}^{x_2} d\psi$

$= \psi(x_1) - \psi(x_2)$

$$\boxed{Q = \psi_2 - \psi_1}$$

Question-9



→ Simplification of continuity equation.

(a) $V_z = v_z(r)$

BC

$p = p_{atm}$ at $r = aR$

$\tau_{rz} = \mu \frac{dV_z}{dr} = 0$ at $r = aR$

$v_z(r) = 0$ at $r = R$

r-momentum: $\frac{\partial p}{\partial r} = 0 \Rightarrow p = p(z)$ only

But $p = p_{atm}$ at $r = aR \Rightarrow p = p_{atm}$ every where inside the [illegible]

$\Rightarrow \frac{\partial p}{\partial z} = 0$

z-momentum: $\mu \frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial V_z}{\partial r}\right] + \rho g = 0$

$\Rightarrow \frac{\partial}{\partial r}\left[r \frac{\partial V_z}{\partial r}\right] = -\frac{\rho g r}{\mu}$

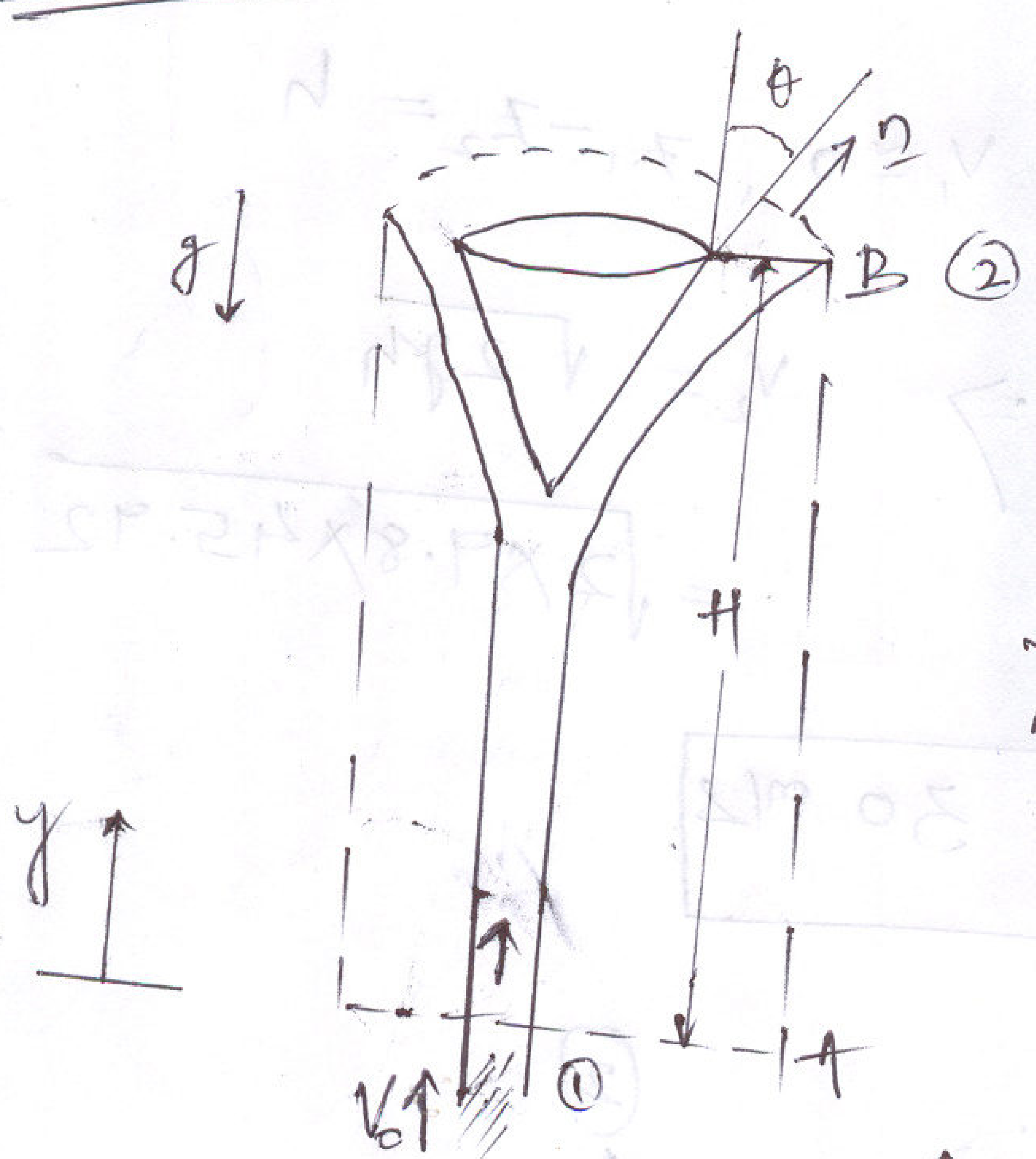
$$\Rightarrow V_z = \frac{-\rho g r^2}{4\mu} + C_1 \ln r + C_2$$

After using BC's

$$V_z = \frac{\rho g R^2}{4\mu}\left[1 - \frac{r^2}{R^2} + 2a^2 \ln\frac{r}{R}\right]$$

(b) $$\tau_{rz}\Big|_{r=R} = \mu \frac{dU_z}{dr}\Big|_{r=R} = \frac{\rho g R}{2}(a^2-1)$$

Question-10



Mass: $V_1 A_1 = V_2 A_2$

y-component Momentum: $F_y = \int_{CS} v_y \rho \underline{v} \cdot d\underline{A}$

$$Mg = \left[V_2 \cos\theta \, \rho V_2 A_2\right] - \left[V_1 \rho V_1 A_1\right]$$

Bernoulli: $V_2^2 = V_1^2 - 2gH$

$V_1 = V_0$;

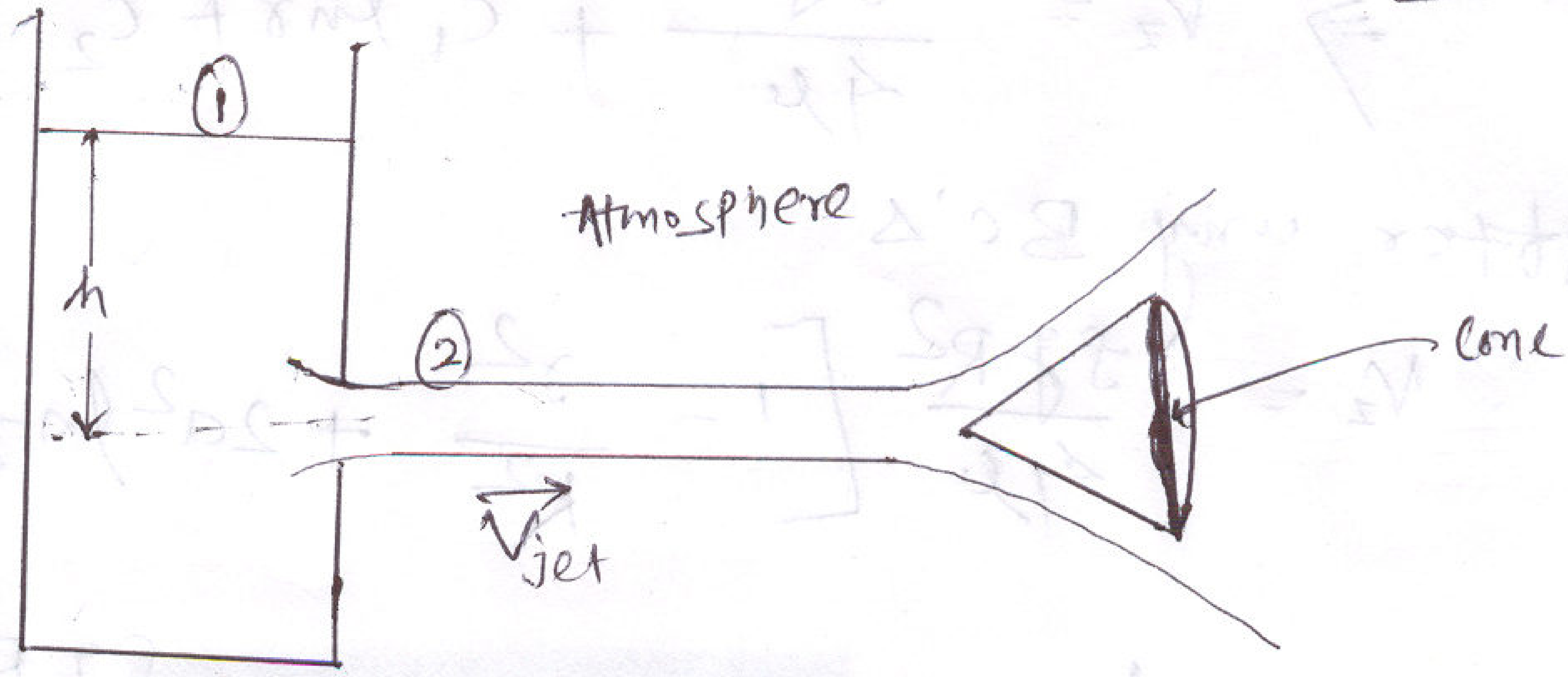
Here
$H = 1\,m$
$V_0 = 10\,m/s$
$D = 100\,mm = 0.1\,m$
$\theta = 30^\circ$

$$\Rightarrow M = \left[\frac{V_0 - \sqrt{V_0^2 - 2gH}\cos\theta}{g}\right]\rho V_0 A_1$$

$$\Rightarrow \boxed{M = 17.9\ kg}$$

Question-11:

(a)



To find the V_{jet}, apply Bernoulli equation between points (1) and (2) which lie on the same streamline.

$$\cancel{\frac{P_1}{\rho}} + gz_1 + \cancel{\frac{V_1^2}{2}} = \cancel{\frac{P_2}{\rho}} + gz_2 + \frac{V_2^2}{2}$$

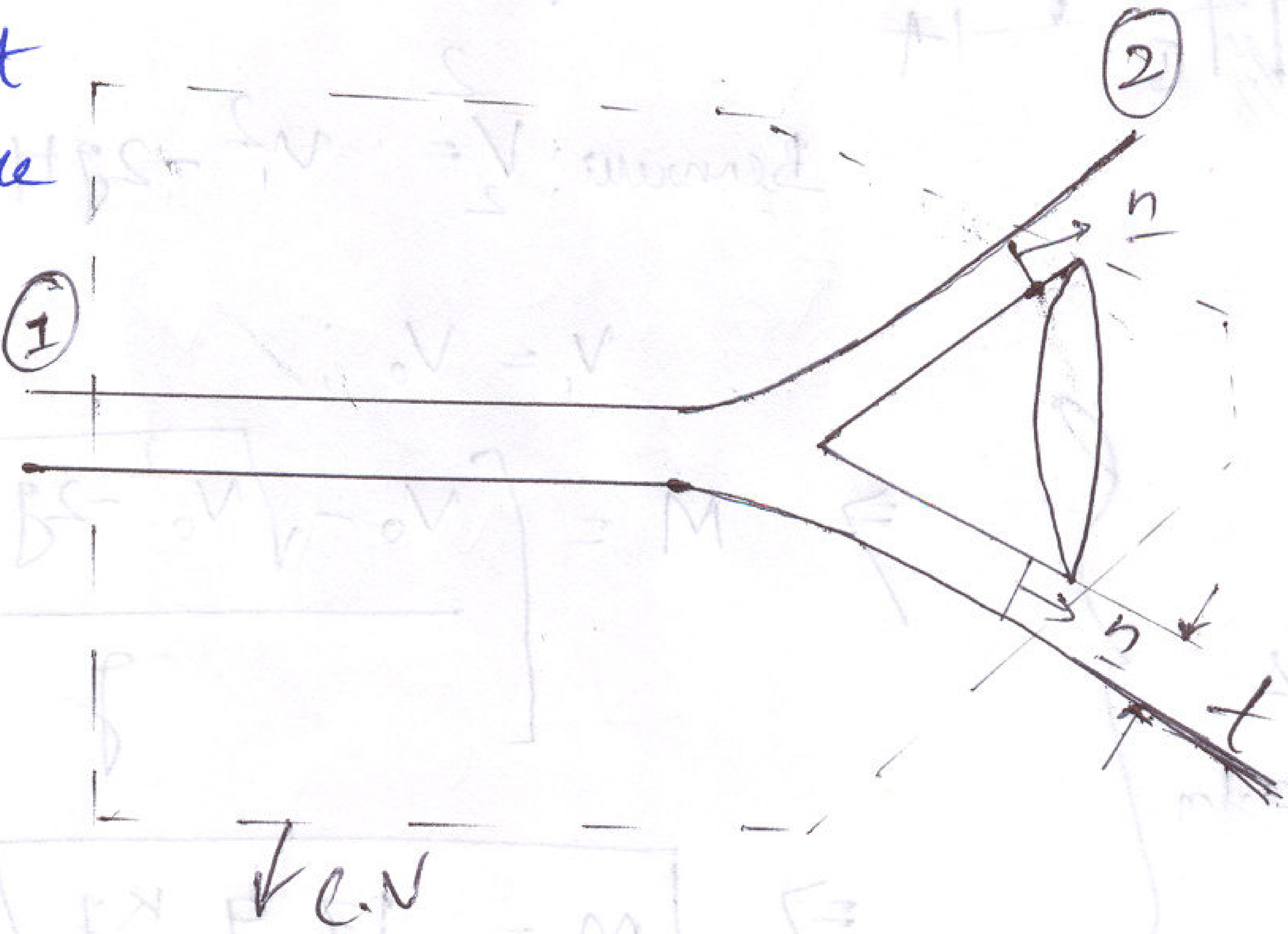
Here $P_1 = P_2 = P_{atm}$, $V_1 \simeq 0$, $z_1 - z_2 = h$

$$\Rightarrow V_2^2 = 2gh \Rightarrow V_2 = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.8 \times 45.92}$$

$$\Rightarrow \boxed{V_{jet} = V_2 = 30 \text{ m/s}}$$

(b) It is convenient to be in the reference frame of the cone.



By mass conservation; $-\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0$

Here $\rho_1 = \rho_2 = \rho$

$$\Rightarrow (V_j + V_c)\frac{\pi D_j^2}{4} = (V_2 + V_c)\, 2\pi R t$$

Area over which flu is leaving normally to the

$$(30+14)\frac{(100\times10^{-3})^2}{4} = 2(V_2+14)\times 230\times10^{-3}\times 5.434\times10^{-}$$

$$\Rightarrow V_2 = 30\ m/s$$ (w.r.t ~~stationary~~ frame of reference)

$V_2 = 44\ m/s$ in the reference frame of the cone.

(c) Momentum balance (for the same C.V shown before)

Steady: $F_x = \int_{C.S} u \rho \underline{v}\cdot\underline{n}$

x-component

$$R_x = u_1(-\rho V_1 A_1) + u_2(\rho V_2 A_2)$$

R_x → External force on the C.V ⇓ force applied on the curve

Here $V_1 = V_2 = V_j$, $A_2 = A_1 = A_j$

$$R_x = -(V_j+V_c)\rho(V_j+V_c)A_j + (V_j+V_c)\cos 60^\circ \rho(V_j+V_c)A_j$$

$$\Rightarrow R_x = \rho(V_j+V_c)^2 A_j(\cos 60^\circ - 1)$$

$$= 10^3(44)^2\frac{\pi}{4}(0.1)^2\left(\frac{1}{2}-1\right)$$

$$= -7602.65\ N$$

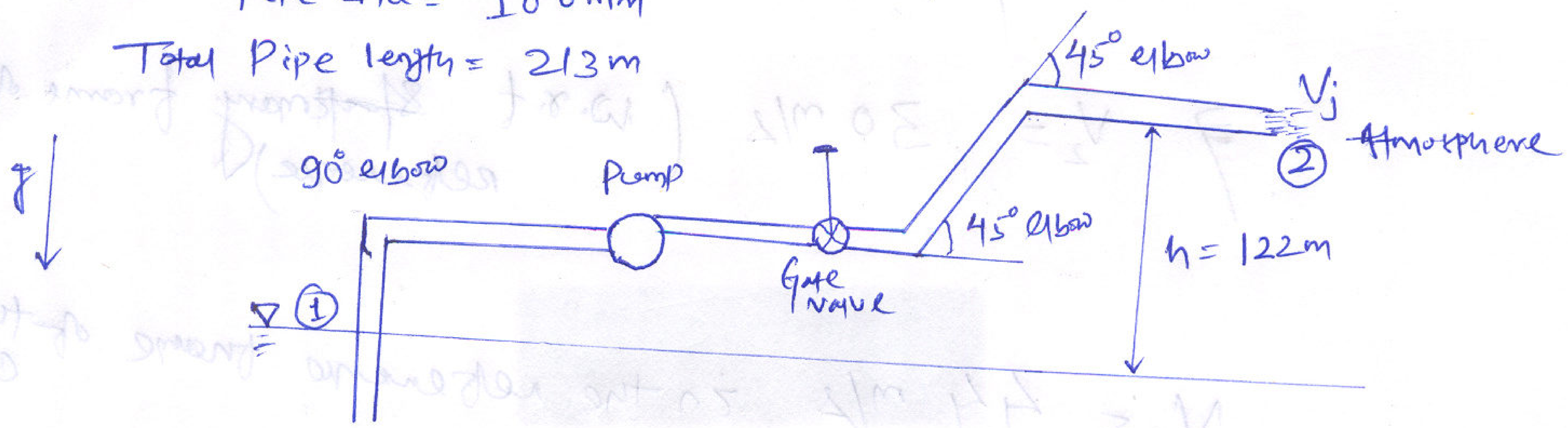
$$= \boxed{-7.6\ KN}$$

So, force must be in the negative x-direction exerted on the Cone $= \boxed{7.6\ KN}$

Question - 12:

Pipe dia = 100 mm

Total Pipe length = 213 m



Minor Loss:
① Entrance
② 90° elbow - 1
③ Gate valve
④ 45° elbow - 2

Applying Energy b-ance between ① and ②

$$\left(\frac{p_1}{\rho} + \alpha\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) + \Delta h_{pump} = h_{lT} \to h_{major}, \; h_{minor}$$

$$h_{major} = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$h_{minor} = \frac{\bar{V}^2}{2}\sum_i K_i$$

$P_1 = P_2 = P_{atm}$, $\bar{V}_1 = 0$; $\bar{V}_2 = 37$ m/s

$\alpha_2 = 1$ (for free jet)

NOTE: $\bar{V}_2$ at the nozzle exit is not the avg $\bar{V}$ in the pipe.

$$\Rightarrow \Delta h_{pump} = gz_2 + \frac{\bar{V}_2^2}{2} + f\frac{L}{D}\frac{\bar{V}^2}{2} + \frac{\bar{V}^2}{2}\left[K_{ent} + K_{90°} + 2K_{45°} + K_{gate}\right]$$

In the Pipe

$$\bar{V} = \frac{Q}{A} = \frac{Q}{\pi D^2/4} = 4.84 \text{ m/s}$$

$$Re = \frac{4.84 \times 100 \times 10^{-3} \times 10^{3}}{10^{-3}}$$

$$Re = 4.84 \times 10^{5}$$

$$\epsilon = 0.0015\,mm$$

$$\Rightarrow \frac{\epsilon}{D} = 1.5 \times 10^{-5}$$

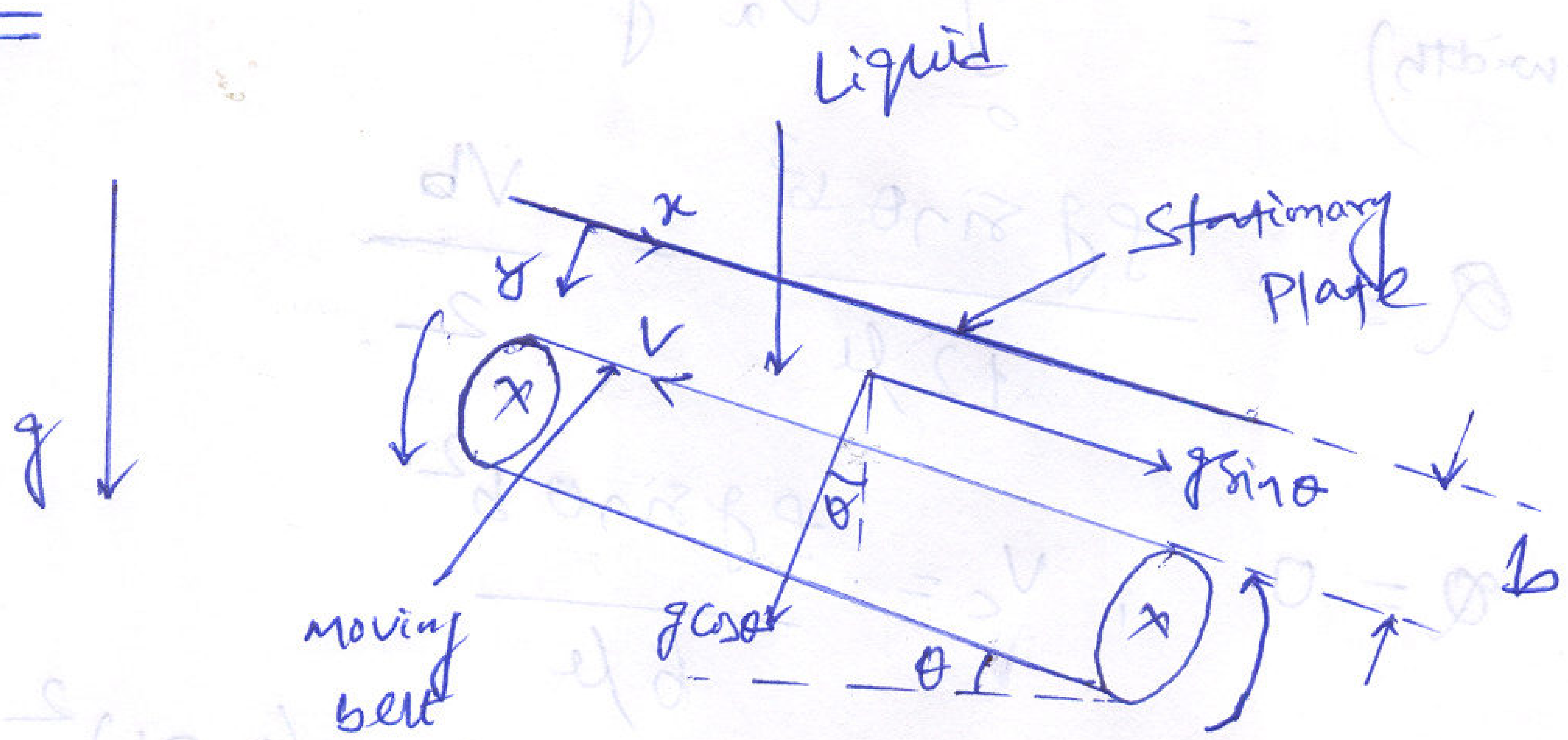
From $f - Re$ chart $\Rightarrow f = 0.0135$

$$\Delta h_{pump} = 2.249 \times 10^{3}\ m^2/s^2$$

$$\text{Power } i/p = \dot{m}\,\Delta h_{pump} = \frac{10^3 \times 38 \times 2.249 \times 10^{3}}{10^{3}}$$

$$\Rightarrow \boxed{\text{Power } i/p \text{ to the pump} = 85.5\ KW}$$

Question-13



(a) Boundary Conditions:

$$\boxed{V_x(y=0) = 0}$$

$$V_x(y=b) = -V$$

(b) x-momentum:

$$\mu \frac{d^2 V_x}{dy^2} + \rho g \sin\theta = 0$$

$$\frac{d^2 V_x}{dy^2} = \frac{-\rho g \sin\theta}{\mu}$$

$$\frac{dV_x}{dy} = \frac{-\rho g \sin\theta}{\mu} y + C_1$$

$$\Rightarrow V_x = \frac{-\rho g \sin\theta}{\mu}\frac{y^2}{2} + C_1 y + C_2$$

Using BC's to get C_1 and C_2 value

$$C_1 = \frac{\rho g \sin\theta}{2\mu} b - \frac{V}{b} \; ; \; C_2 = 0$$

$$V_x(y) = \frac{\rho g \sin\theta\, b^2}{2\mu}\left[\frac{y}{b} - \frac{y^2}{b^2}\right] - \frac{V}{b} y$$

$$\underset{(\text{Per unit width})}{Q} = \int_0^b V_x\, dy$$

$$Q = \frac{\rho g \sin\theta\, b^3}{12\mu} - \frac{Vb}{2}$$

if $Q = 0$, $V_c = \frac{\rho g \sin\theta\, b^2}{6\mu}$

$$= \frac{10^3 \times 9.8 \times \frac{1}{\sqrt{2}} \times (0.01)^2}{6 \times 0.1}$$

$$\Rightarrow \boxed{V_c = 1.155\ m/s}$$

Question-14:

(a) Euler equation: (Steady)

$$\rho(\underline{V}\cdot\nabla)\underline{V} = -\nabla P + \rho\underline{g}$$

Vector identity $(\underline{V}\cdot\nabla)\underline{V} = \nabla(\frac{1}{2}\underline{V}\cdot\underline{V}) + \underbrace{(\nabla\times\underline{V})}_{\underline{\omega}}\times\underline{V}$

$$\Rightarrow \left[\nabla(\tfrac{1}{2}\underline{V}\cdot\underline{V}) + \underline{\omega}\times\underline{V} + \tfrac{1}{\rho}\nabla P - \underline{g}\right]\cdot d\underline{r} = 0$$

Along a streamline, $(\underline{\omega}\times\underline{V})\cdot d\underline{r} = 0$

$$\underline{g} = -g\hat{k}$$

$$\Rightarrow \nabla(\tfrac{1}{2}\underline{V}\cdot\underline{V})\,d\underline{r} + \tfrac{1}{\rho}\nabla P\cdot d\underline{r} - \underline{g}\cdot d\underline{r} = 0$$

$$\Rightarrow d(\tfrac{1}{2}\underline{V}\cdot\underline{V}) + d(P/\rho) + g\,dz = 0$$

$\Rightarrow$ Along a streamline

$$d\left[\frac{1}{2}V^2 + \frac{P}{\rho} + gz\right] = 0$$

$$\Rightarrow \frac{V^2}{2} + \frac{P}{\rho} + gz = \text{Constant along a streamline}$$

(b) Streamlines $\Rightarrow$ Constant $\psi \Rightarrow d\psi = 0$

$$d\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy = 0$$

Slope of a streamline $\Rightarrow \left.\frac{dy}{dx}\right|_\psi = \frac{-\partial\psi/\partial x}{\partial\psi/\partial y} = \frac{V}{u}$

Equipotential $\Rightarrow$ Constant $\phi \Rightarrow d\phi = 0$

$$d\phi = u\,dx + V\,dy = 0$$

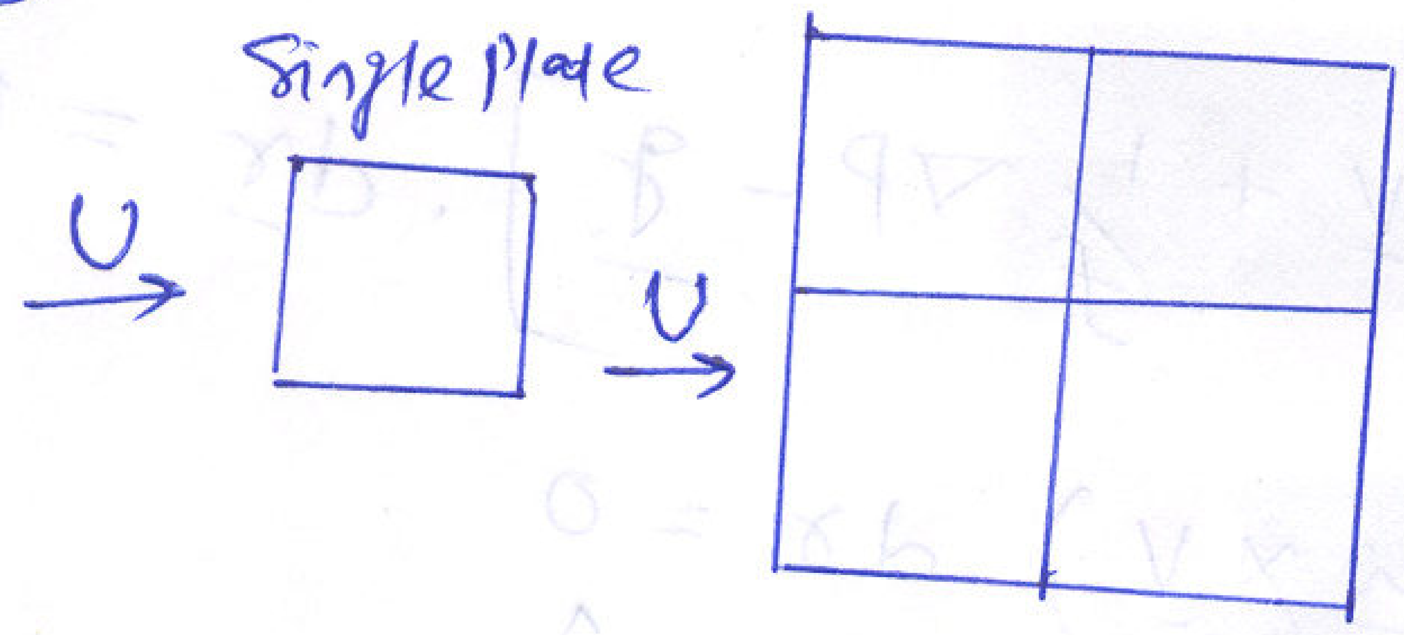
$$\left.\frac{dy}{dx}\right|_\phi = -u/V$$

Multiplying the two slopes :

$$\frac{V}{u} \times \left(-\frac{u}{V}\right) = -1 \Rightarrow$$ Streamlines of equipotential are orthogonal.

Question-15 :

(a)



$$C_f = \frac{C_1}{\sqrt{Re_L}}$$

$$F_A = \cancel{\frac{C_1}{\sqrt{2L}}} \quad F_A = \frac{C_1}{\sqrt{2L_1}\sqrt{\frac{V\rho}{\mu}}} 4A$$

$$F_B = \frac{C_1}{\sqrt{4L_1}} \cdot \frac{1}{\sqrt{\frac{V\rho}{\mu}}} 4A_1$$

$$\Rightarrow F_A = \sqrt{8} F_1 \text{ and } F_B = 2F_1$$

$$\Rightarrow \boxed{F_A > F_B}$$

(b)

Density of napthalene at the surface $= \rho_{A0}$

$$PV = \frac{m}{M} RT$$

$$\rho_{A0} = \frac{m}{V} = \frac{P_A M}{RT} = \frac{\left(\frac{1}{760} \times 1.013 \times 10^5 \text{ Pa}\right) 128}{8314 \times 300}$$

$$\Rightarrow \rho_{A0} = 6.8 \times 10^{-3} \text{ Kg/m}^3$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \rho_A}{\partial r}\right) = 0$$

$$\Rightarrow \rho_A = \frac{c_1}{r} + c_2$$

B C's

$\rho_A (r = R_1) = \rho_{A0}$

$\rho_A (r \to \infty) = 0$

$$\Rightarrow \rho_A = \frac{\rho_{A0}}{r} R_1$$

$$\text{Flux} = -D_{AB} \left.\frac{\partial \rho_A}{\partial r}\right|_{r=R_1} = \frac{D_{AB}\rho_{A0}}{R_1^2}$$

Rate of evaporation = Flux . Area

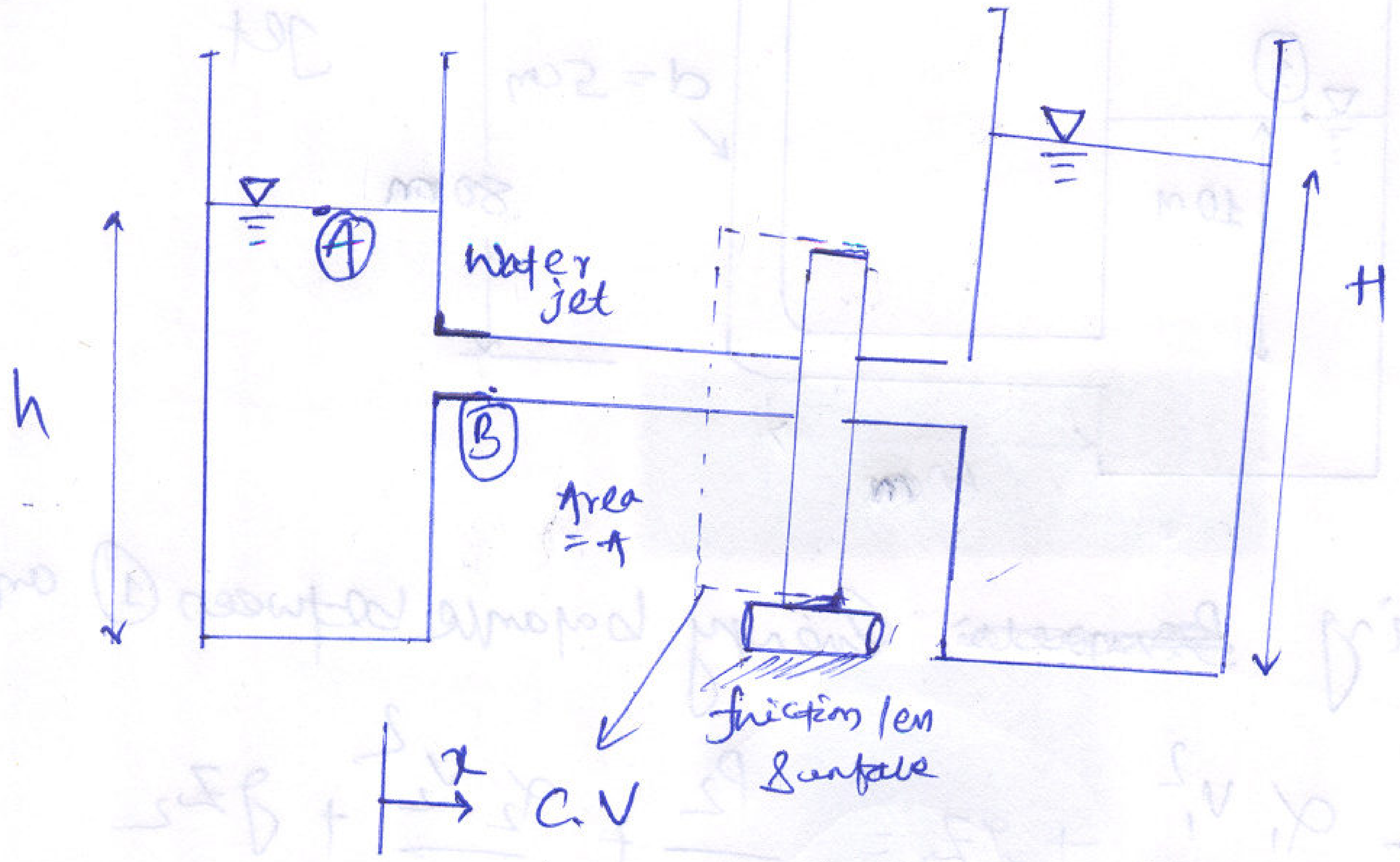
$$= \frac{D_{A} \rho_{A0}}{R_1} \cdot 4\pi R_1^2$$

$$= 4\pi R_1 D_{AB} \rho_{A0}$$

$$= 4\pi \left(\tfrac{1}{2} \times 10^{-2}\right) \times \left(5 \times 10^{-6}\right) \times \left(6.8 \times 10^{-3}\right)$$

$$\boxed{\text{Rate of evaporation} = 2.136 \times 10^{-9}\ \text{kg/s}}$$

Question-16:



(Steady) Integral momentum balance:

$$\underline{F} = \int \rho \, \underline{v} \, \underline{v} \cdot \underline{n} \, dA$$

x-component: (uniform flow)

$$F_x = \int_{C.S} \rho V_x (-V_x) \, dA \qquad \Big| \; V_x = V_{jet}$$

$$= -\rho V_x^2 A \qquad \Big| \; F_x = -\rho g H$$

$$-\rho g H A = -\rho V_{jet}^2 A \quad \text{— (I)}$$

By applying Bernoulli equation between point (A) and (B)

$$\Rightarrow \frac{P_A}{\rho} + \frac{V_A^2}{2} + gZ_A = \frac{P_B}{\rho} + \frac{V_B^2}{2} + gZ_B \quad \text{— (II)}$$

Here $P_A = P_B = P_{atm}$; $V_A \simeq 0$, $Z_A - Z_B = h$

$V_B = V_{jet}$

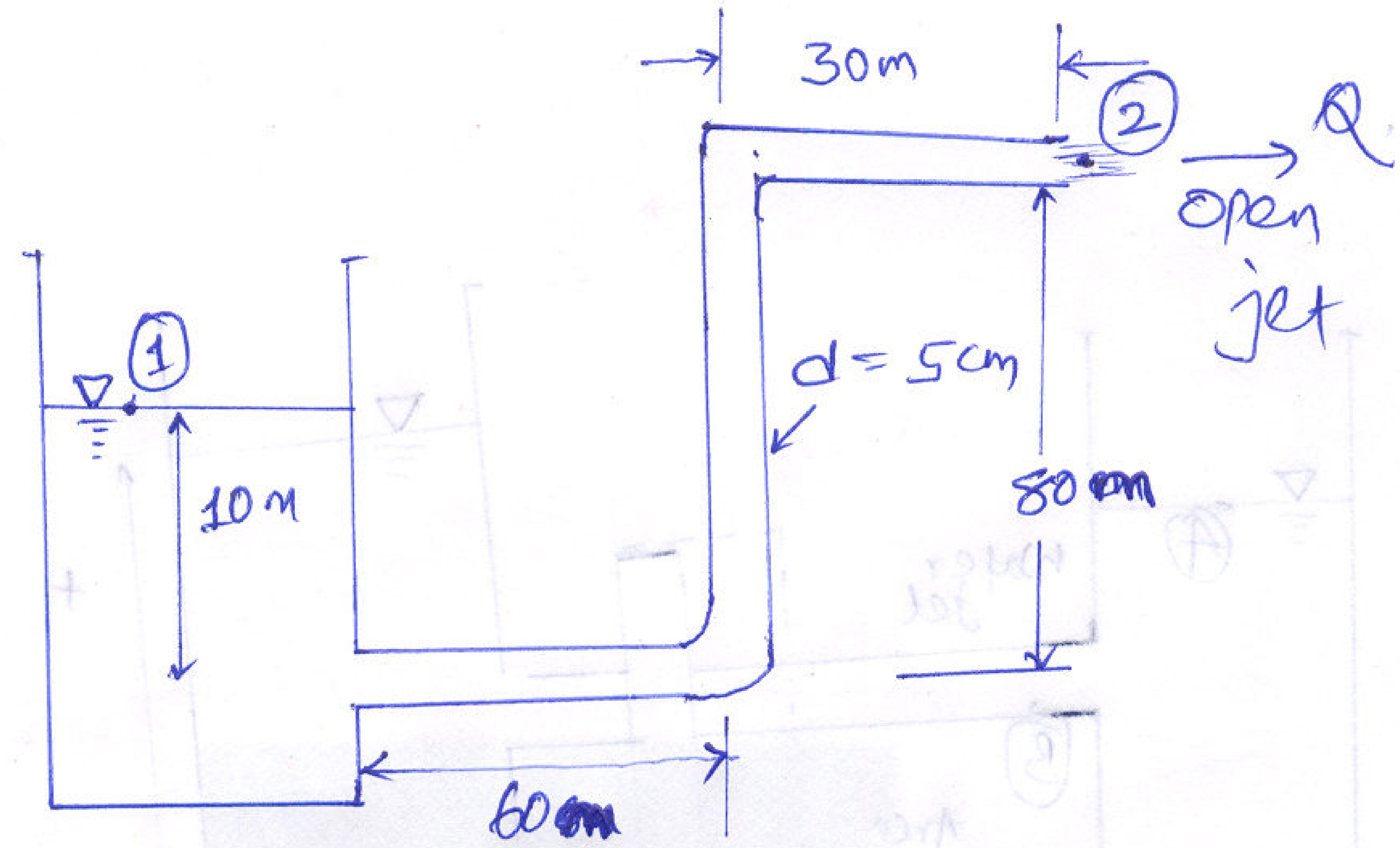
$\Rightarrow$ Bernoulli equation gives

$$V_{jet}^2 = 2gh \quad \text{— (III)}$$

Put eq^n-(III) in eq^n-(I), then

$$gH = V_{jet}^2 \Rightarrow gH = 2gh \Rightarrow \boxed{h = \frac{H}{2}}$$

Question-17 :



Applying ~~Bernoulli~~ Energy balance between 1 and 2

$$\frac{P_1}{\rho} + \frac{\alpha_1 V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{\alpha_2 V_2^2}{2} + gZ_2 + f\frac{L}{D}\frac{V_2^2}{2g} + \sum_i K_i \frac{V_2^2}{2g}$$

Here $P_1 - P_2 = P_{1g}$; $V_1 \approx 0$

$V_2 = V_{pipe}$ (by mass balance)

$Z_1 = 10m,\ Z_2 = 80m$

$$\Rightarrow \frac{P_{1g}}{\rho} = \frac{V^2}{2}\left[\alpha_2 + f\frac{L}{D} + K_{ent} + 2K_{90}\right] + g.70$$

$$Q = 60\ \frac{m^3}{hr} = \frac{1}{60}\ m^3/s$$

$$V = \frac{Q}{A} = \frac{1}{60}\cdot\frac{4}{\pi(0.05)^2} = 8.49\ m/s$$

$$\Rightarrow Re \text{ in the pipe} = \frac{\rho V D}{\mu} = \frac{10^3 \times 8.49 \times 0.05}{10^{-3}} = 4.245 \times 10^5$$

$\Rightarrow$ Flow is turbulent $\Rightarrow$ From f-Re Chart

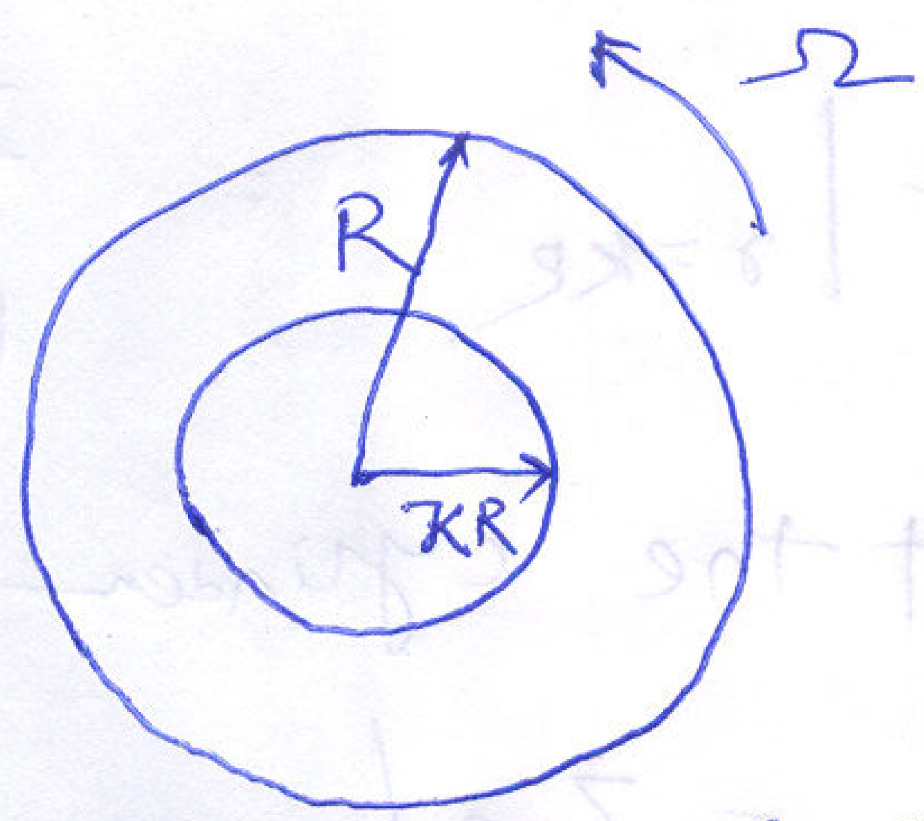
$\Rightarrow \boxed{f \approx 0.0136}$

So, $P_{1g} = \frac{\rho v^2}{2}\left[1 + \frac{0.0136 \times 70}{0.05} + 0.5 + 2 \times 0.95\right] + 70\rho g$

$= 2.475 \times 10^6 \ Pa$

$$\boxed{P_{1g} = 2.475 \ MPa}$$

Question-18



Here, only non zero velocity component $v_\theta(r)$

Steady, axisymmetric flow in θ-direction.

$v_\theta, p \Rightarrow$ indep of θ

$$\mu \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right] = 0$$

$$\Rightarrow \frac{\partial}{\partial r}(r v_\theta) = c_1 r$$

$$\Rightarrow r v_\theta = c_1 \frac{r^2}{2} + c_2$$

or $$\boxed{v_\theta = c_1 \frac{r}{2} + \frac{c_2}{r}}$$

B.C's

$v_\theta(r = KR) = 0$

$v_\theta(r = R) = \Omega R$

$$V_\theta = \frac{\Omega R\left[\frac{r}{R} - \frac{\kappa^2 R}{r}\right]}{(1-\kappa^2)}$$

$$\tau_{r\theta} = \mu r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)$$

$$\tau_{r\theta} = \frac{\mu \Omega R}{\left(\frac{1}{\kappa} - \kappa\right)} \cdot \frac{2\kappa R}{r^2}$$

Stress on the inner cylinder at $r = \kappa R$

$$\tau_{r\theta}\Big|_{r=\kappa R} = \frac{2\mu\Omega}{(1-\kappa^2)}$$

Torque at the cylinder $(r = \kappa R)$

$$= \tau_{r\theta}\Big|_{r=\kappa R} \times 2\pi\kappa R L \times \kappa R$$

$$\Rightarrow \boxed{\text{Torque on the inner cylinder} = \frac{4\pi\mu\Omega\kappa^2 R^2 L}{1-\kappa^2}}$$

Question-19:

(a) Potential flow:

$$\phi = A r^2 \cos 2\theta$$

$$v_r = \frac{\partial\phi}{\partial r} = 2Ar\cos 2\theta$$

$$v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -2Ar\sin 2\theta$$

$$v_\theta = -\frac{\partial\psi}{\partial r}\ ; \quad v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = 2Ar\cos 2\theta$$

$$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = 2Ar\cos 2\theta$$

$$\Rightarrow \psi = Ar^2 \sin 2\theta + C_1(r)$$

$$-\frac{\partial \psi}{\partial r} = -2Ar\sin 2\theta$$

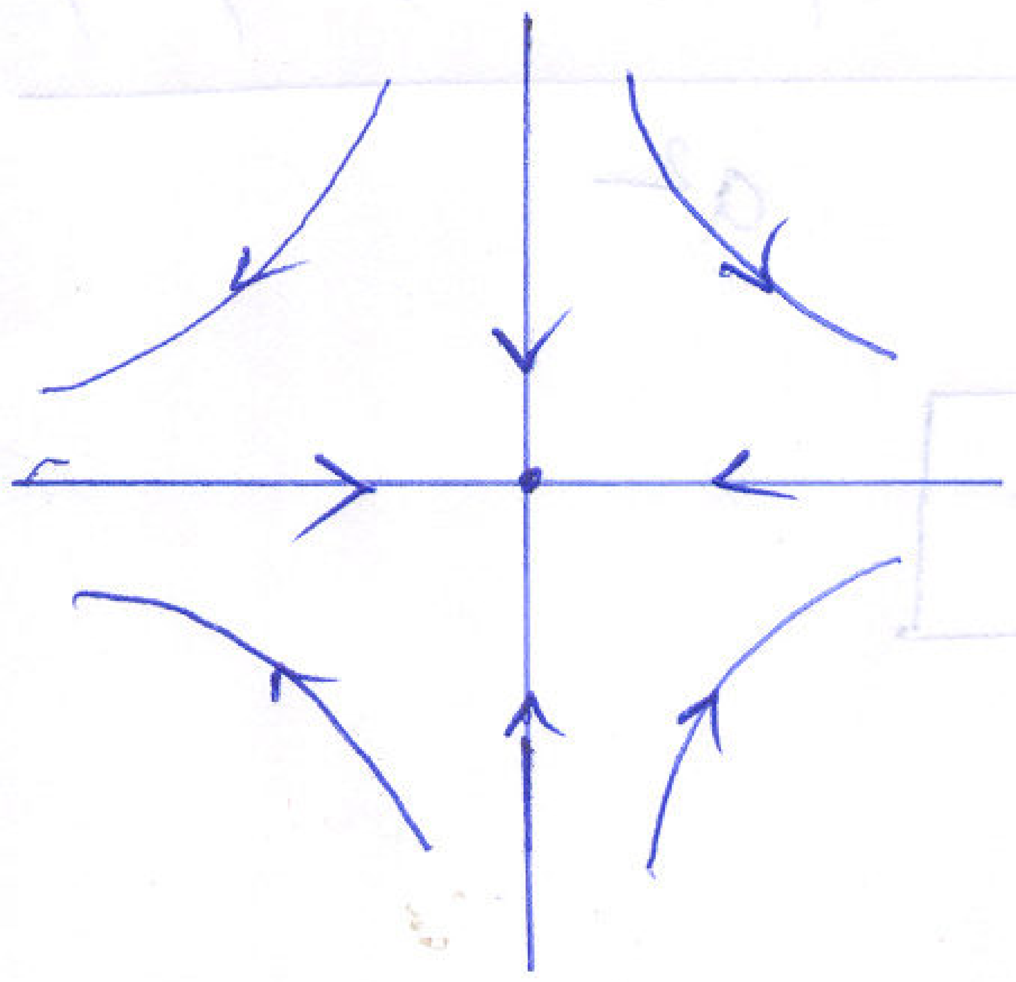
$$\Rightarrow \psi = Ar^2 \sin 2\theta + C_2(\theta)$$

$$\Rightarrow \psi = Ar^2 \sin 2\theta$$

Consider the stream line $\psi = 0$

$$\Rightarrow \sin 2\theta = 0 \quad \Rightarrow 2\theta = n\pi$$

$$\Rightarrow \theta = \frac{n\pi}{2}, \quad n = 0, 1, 2 \ldots$$



Flow near a stagnation point

(b) $$C_f = \frac{0.664}{Re_x^{1/2}}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\left(\frac{\rho U}{\mu}\right)^{1/2} x^{1/2}}$$

$$\frac{\tau_w(x)}{\frac{1}{2}\rho U^2} = \left(\frac{\mu}{\rho U}\right)^{1/2} \frac{0.664}{x^{1/2}}$$

Force on a single plate $= a\int_0^L \tau_w(x)\,dx$

$$= \frac{1}{2}\rho U^2 \times a\left(\frac{\mu}{\rho U}\right)^{1/2} \times 0.664\int_0^L \frac{dx}{x^{1/2}}$$

$$= 0.664 \times a \times \left(\frac{\mu}{aV}\right)^{1/2} \times \sqrt{L} \times \rho U^2$$

$$= 0.664 \times a \times \mu^{1/2} \times \rho^{1/2} \times U^{3/2}\, L^{1/2}$$

F on one plate $= 0.664 \times (\rho\mu L)^{1/2} \times a \times U^{3/2}$

F on four plates $= 4 \times 0.664\,(\rho\mu L)^{1/2} \times a \times U^{3/2}$

By an integral momentum balance:

$$\Delta P = \frac{F}{a^2} = \frac{4 \times 0.664 \times (\rho\mu L)^{1/2} \times a \times U^{3/2}}{a^2}$$

$$\Rightarrow \boxed{\Delta P = 6.17\ Pa}$$